Discontinuous Galerkin spectral element methods for earthquake simulations

Paola F. Antonietti

MOX, Dipartimento di Matematica, Politecnico di Milano

Séminaire du Laboratoire Jacques-Louis Lions
Université Pierre et Marie Curie, Paris, September, 23 2016
Goals

Simulation of **large-scale seismic events** at regional scale: from far-field to near-field including soil-structure interaction effects

- Seismic hazard analysis
- Dynamic analysis of structural systems
- Plan of protection strategies and damage loss
People

A. Quarteroni (MOX, PoliMi & EPFL)
R. Paolucci (DICA, PoliMi)
M. Stupazzini (MunichRE)
I. Mazzieri (MOX, PoliMi)
A. Ferroni (MOX, PoliMi)

C. Smerzini (DICA, PoliMi)
A. Ozcebe (DICA, PoliMi)
K. Hashemi (DICA, PoliMi)
R. Guidotti (U. Illinois)
F. Gatti (DICA, PoliMi & Ecole Centrale Paris Suplec)
Seismic waves

Seismic waves are **propagating vibrations** that carry **energy** from the source of the shaking outward in all directions.

M6.3 - L’aquila, 2009-04-06 01:32:39
(source: earthquake.usgs.gov)

Italian earthquake potential hazard, 2015
(source: Italian Civil Defence)
Seismic waves

- Seismic waves
  - Compressional or P (primary)
  - Transverse or S (secondary)
  - Love
  - Rayleigh

- An earthquake radiates P and S waves in all directions.

- The interaction of the P and S waves with Earth’s surface → surface waves.

- P and S waves travel at different speeds (used to locate earthquakes)
Seismic waves (cont’d)

P-Waves (Compressional)
- The ground is vibrated in the direction the wave is propagating.
- Travel through all types of media.
- \( C_P = \sqrt{(\lambda + 2\mu)/\rho} \) (P-wave velocity)
- Typical speed: \( \sim 1 \rightarrow 14 \text{ km/sec} \)

S-Waves (Transverse)
- The ground is vibrated in the perpendicular direction to that the wave is propagating.
- Travel only through solid media.
- \( C_S = \sqrt{\mu/\rho} \) (S-wave velocity)
- Typical speed: \( \sim 1 \rightarrow 8 \text{ km/sec} \)
SPEED: S-Pectral Elements in Elastodynamics with Discontinuous Galerkin

- Open-source Fortran code for 3D seismic wave propagation
- Discontinuous Galerkin Spectral Element Code
- Parallel kernel with hybrid OpenMP-MPI programming
- Emerging application within the European project PRACE-2IP WP 9.3
Applications

Dinamic Soil Structure Interaction

Traffic Induced Vibrations

Seismic Wave Propagation in Complex Geological Configurations
Requirements on the numerical scheme

- **FLEXIBILITY**
  Highly heterogeneous materials and complex geometries

- **ACCURACY**
  Avoid dispersive effects on the propagating wave field
  Reproduce topographic effects and soil amplifications

- **EFFICIENCY**
  Scalable implementation in parallel machines

Discontinuous Galerkin Spectral Element methods

- [Kaser, Dumbser, 2006], [Chung, Enquist, 2006], [Riviere, Shaw, Whiteman, 2007],
  [de Basabe, Sen, Wheeler, 2008], [Grote, Diaz, 2009], [Wilcox, Stadler, Burstedde, Ghattas, 2010],
  [A., Mazzieri, Rapetti, Quarteroni, 2012], [Etienne, Chaljub, Virieux, Glinsky, 2010],
  [Mazzieri, Stupazzini, Smerzini, Guidotti, 2013], [Peyrusse, Glinsky, Gélis, Lanteri, 2014],
  [A., Ayuso, Mazzieri, Quarteroni, 2015], [A., Marcati, Mazzieri, Quarteroni, 2015]
  [Paolucci, Mazzieri, Smerzini, 2015], [A., Dal Santo, Mazzieri, Quarteroni, 2016], [Ferroni, A., Mazzieri, ...]
The mathematical model

Let $\Omega \subset \mathbb{R}^d$, $d = 2, 3$ with boundary $\partial \Omega = \Gamma_D \cup \Gamma_N$ s.t. $|\Gamma_D| > 0$. The mathematical model of linear elastodynamics reads:

$$\rho(x)u_{tt}(x, t) - \nabla \cdot \sigma(x, t) = f(x, t), \quad \text{in } \Omega \times (0, T],$$

$$u(x, t) = 0, \quad \text{on } \Gamma_D \times (0, T],$$

$$\sigma(x, t)n(x) = g(x, t), \quad \text{on } \Gamma_N \times (0, T],$$

$$u_t(x, 0) = u_1(x), \quad \text{in } \Omega \times \{0\},$$

$$u(x, 0) = u_0(x), \quad \text{in } \Omega \times \{0\},$$

Visco-elastic forces can be introduced in term of $f_\xi(u, u_t) = -2\rho\xi u_t - \rho\xi^2 u$, ($\xi$ decay factor).
The mathematical model: notation

- \( u : \Omega \times [0, T] \rightarrow \mathbb{R}^d \) is the displacement vector field.
- \( \varepsilon(u) = \frac{1}{2} (\nabla u + \nabla u^T) \) is the strain tensor.
- \( \sigma : \Omega \times [0, T] \rightarrow S = \mathbb{R}^{d \times d}_{\text{sym}} \) is the Cauchy stress tensor (Hooke’s law).
  \[
  \sigma(x, t) = D\varepsilon(u(x, t))
  \]
- \( D : S \rightarrow S \) is the stiffness tensor (spd, uniformly bounded).
  \[
  D\tau = 2\mu\tau + \lambda\text{tr}(\tau)I \quad \forall \tau \in S
  \]
- \( \lambda, \mu \in L^\infty(\Omega) \) are the Lamé parameters.
- \( \rho \in L^\infty(\Omega) \) is the mass density.
- \( C_P = \sqrt{(\lambda + 2\mu)/\rho} \) (P-wave velocity), \( C_S = \sqrt{\mu/\rho} \) (S-wave velocity).
Weak form

For any \( t \in (0, T] \) find \( \mathbf{u} = \mathbf{u}(t) \in H_{0, \Gamma_D}^1(\Omega) \) such that:

\[
(\rho \mathbf{u}_{tt}, \mathbf{v})_\Omega + (\mathcal{D} \varepsilon(\mathbf{u}), \varepsilon(\mathbf{v}))_\Omega = (f, \mathbf{v})_\Omega + (g, \mathbf{v})_{\Gamma_N} \quad \forall \mathbf{v} \in H_{0, \Gamma_D}^1(\Omega).
\]

The above problem is well posed, and its unique solution satisfies a stability estimate in the following energy norm

\[
\| \mathbf{u}(s) \|_E^2 = \| \rho^{1/2} \mathbf{u}_t(s) \|_{L^2(\Omega)}^2 + \| \mathcal{D}^{1/2} \varepsilon(\mathbf{u})(s) \|_{L^2(\Omega)}^2 \quad \forall s \in [0, T].
\]

For the proof see [Raviart, Thomas, 1983], [Duvaut, Lions, 1976].
SPEED paradigm

1st - LEVEL
- Subdivide $\Omega$ into macroregions $\Omega_k$’s.
- Define a polynomial approximation degree $p_k \geq 1$ in each $\Omega_k$.

2nd- LEVEL
- Introduce a (hexahedral/tetrahedral) conforming partition $T_h^k$ inside each $\Omega_k$ (non-matching at the interfaces)

3rd - LEVEL
- Spectral Element Method (SEM) / Tetrahedral Spectral Element Method (TSEM) within in each $\Omega_k$
v ∈ V_{hp} is

- discontinuous across the skeleton \( \mathcal{F}_h \)
- continuous inside each \( \Omega_k \)
- a (suitable) polynomial of degree \( p_k \) when restricted to a mesh element \( \kappa ∈ \mathcal{T}_h^k \)
SPEED paradigm

- SEM: classical Lagrangian shape functions associated to the GLL points
- TSEM: $C^0$ boundary adapted basis [Sherwin et al., 2005] (modification of the basis of [Dubiner, 1991])
- If the first two levels coincide $\iff$ elementwise DG approach
Weighted average and jump trace operators

For (regular-enough) vector-valued and tensor-valued functions $v$ and $\tau$ define on each $F \in \mathcal{F}_h$ on the skeleton shared by elements $\kappa^\pm \in \Omega^\pm$

$$\{v\}_\delta = \delta v^+ + (1 - \delta)v^-,$$
$$\{\tau\}_\delta = \delta \tau^+ + (1 - \delta)\tau^-,$$  \hspace{1cm} $\delta \in [0, 1]$

$$[v] = v^+ \odot n^+ + v^- \odot n^-,$$
$$[\tau] = \tau^+ n^+ + \tau^- n^-,$$

- Here $v \odot n = (vn^T + nv^T)/2$
- With the above definitions $[v] \in S = \mathbb{R}^{d \times d}_{sym}$
- If boundary conditions are enforced "à la DG", the above definition modifies accordingly on Dirichlet boundary faces
For any $t \in (0, T]$, find $u^h = u^h(t) \in V_{hp}$ such that

$$\left( \rho u^h_{tt}, v \right)_{\Omega_h} + A(u^h, v) = (f, v)_{\Omega_h} + \left\langle g, v \right\rangle_{\mathcal{F}_h} \quad \forall v \in V_{hp}$$

$$A(w, v) = \sum_k (\varepsilon(w), D\varepsilon(v))_{\Omega_k} - \left\langle \{D\varepsilon(w)\} \delta, [v] \right\rangle_{\mathcal{F}_h}$$

$$\gamma = \alpha\{D\} \frac{\max(p_{\kappa^+}^2, p_{\kappa^-}^2)}{\min(h_{\kappa^+}, h_{\kappa^-})} \quad \alpha \text{ (large enough) positive constant}$$

- **Symmetric Interior Penalty** method (SIP): $\delta = 0.5$ [Wheeler, 1978], [Arnold, 1982]

- **Weighted Symmetric Interior Penalty** method (wSIP): $\delta \neq 0.5$ [Stenberg, 1998]

[Perugia, Schotzau, 2001], [Houston, Schwab, Suli, 2002], [....]
Displacement-stress DG formulations

Let $\Sigma_h = [V_{hp}]^d$. Find $(u^h, \sigma^h) \in C^2((0, T]; V_{hp}) \times C^0((0, T]; \Sigma_h)$ such that

$$
\begin{align*}
(\rho u^h_{tt}, v)_{T_h} + (\sigma^h, \varepsilon(v))_{T_h} - \langle \{\sigma^h\} \delta, [v] \rangle_{\mathcal{F}_h} &= (f, v)_{T_h} + \langle g, v \rangle_{\mathcal{F}_h}, \\
(A\sigma^h - \varepsilon(u^h), \tau)_{T_h} - \langle \{u^h\}(1-\delta), [\tau] \rangle_{\mathcal{F}_h} + \langle c_{22} [\sigma^h], [\tau] \rangle_{\mathcal{F}_h} &+ \langle [u^h], \{\tau\} \rangle_{\mathcal{F}_h} + \langle c_{22} \sigma^h n, \tau n \rangle_{\mathcal{F}_h} = \langle c_{22} g, \tau n \rangle_{\mathcal{F}_h}.
\end{align*}
$$

for all $v \in V_{hp}$ and $\tau \in \Sigma_h$

- **Full discontinuous Galerkin (FDG) method**: $c_{22} \neq 0$, $c_{11} \neq 0$, $\delta = 0.5$ [Castillo, Cockburn, Perugia, Schotzau, 2000]

- **Local discontinuous Galerkin (LDG) method**: $c_{22} = 0$, $\delta = 0.5$ [Cockburn, Shu, 1989]

- **Alternating choice of fluxes (ALT) method**: $c_{22} = c_{11} = 0$ and $\delta = 1$ or $\delta = 0$ [Cheng, Shu, 2008], [Xu, Shu, 2012]
Stability analysis

Let $\alpha$ be large enough and let $u^h \in V_{hp}$ be the approximate solution obtained with a *displacement* DG method. Then

$$
\|u^h(t)\|_\varepsilon \lesssim \|u^h(0)\|_\varepsilon + \int_0^t \rho_*^{-1}\|f(\tau)\|_{L^2(\Omega)} \, d\tau \quad 0 < t \leq T.
$$

$$
\|u(s)\|_\varepsilon^2 = \|\rho^{1/2}u_t(s)\|_{L^2(\Omega)}^2 + \|u(s)\|_{DG}^2
$$

- **Unified derivation and analysis** for the elementwise DG approach using the *flux formulation* [Arnold, Brezzi, Cockburn, Marini, 2001/02]

- Here, $g = 0$, for simplicity

- Analogous stability bounds hold for *displacement-stress* DG methods (for ALT method provided that $\Gamma_N = \emptyset$)

- In the absence of external forces, *displacement-stress* DG methods are fully conservative

♣ [A., Ayuso, Mazzieri, Quarteroni, 2015], [A., Ferroni, Mazzieri, Quarteroni, 2016]
Semi-discrete a-priori error estimates

Assume that $\alpha$ is large enough. Let $u^h$ be the approximated solution of $u$, assumed sufficiently regular. Then

$$\sup_{0<s\leq T} \| u(s) - u^h(s) \|_E \lesssim \frac{h^{m-1}}{p^{k-3/2}}$$

where $m = \min(p+1, k)$ with $k > 1 + d/2$. The hidden constant depends on $C(u, u_t, u_{tt}, T, \text{material properties})$.

$$\| u(s) \|_E^2 = \| \rho^{1/2} u_t(s) \|_{L^2(\Omega)}^2 + \| u(s) \|_{DG}^2 + \| \gamma^{-1/2} \{D\varepsilon(u)\} \|_{L^2(F_h)}^2$$

- No need to penalize the jumps of $u_t$
- Optimal rate of convergence with respect to $h$
- The same result holds for displacement-stress DG formulations (in a suitable energy norm)

♣ [A., Ayuso, Mazzieri, Quarteroni, 2015], [A., Ferroni, Mazzieri, Quarteroni, 2016]
From standard-shaped elements to polytopic grids

- Polyhedral grids provide extreme flexibility in complex geometries
- DG methods are naturally well suited to handle polytopic grids
- Employ an elementwise DG approach in the region where polytopic elements appear
- Assume (for simplicity) that $p_\kappa = p \geq 1$, for any $\kappa$. 
Polytopic grids

- Element interfaces $F$ with **arbitrarily small measure**
- On polytopic elements the discrete space of polynomials of degree $p$ is built on the physical frame.
DG formulation on polytopic grids

For any $t \in (0, T)$, find $u^h = u^h(t) \in V_{hp}$ such that

$$(\rho u^h_{tt}, v)_{\mathcal{T}_h} + A(u^h, v) = (f, v)_{\mathcal{T}_h} + \langle g, v \rangle_{\mathcal{F}_h} \quad \forall v \in V_{hp}$$

The bilinear form associated to the interior penalty DG method reads as before

$$A(w, v) = \sum_k (\varepsilon(w), D\varepsilon(v))_{\Omega_k} - \langle \{D\varepsilon(w)\}^\delta, [v] \rangle_{\mathcal{F}_h}$$

$$- \langle [[w]], \{D\varepsilon(v)\}^\delta \rangle_{\mathcal{F}_h} + \langle \gamma[[w]], [v] \rangle_{\mathcal{F}_h}$$

$$\gamma(x) = \alpha \{D^{1/2}\} \max_{\kappa \in \{\kappa^+, \kappa^\pm\}} \left\{ C_{INV} \frac{p^2|F|}{|\kappa|} \right\}, \quad x \in F, F \in \mathcal{F}_h$$

with $\alpha > 0$ large enough (independent of $p$, $|F|$ and $|\kappa|$).

[\text{[A. Brezzi, Marini, 2009], [Bassi et al, 2014, 2014], [A., Giani, Houston, 2013], [Cangiani, Georgoulis, Houston, 2014], [Cangiani, Dong, Georgoulis, Houston, M2AN, 2015], [A., Houston, Sarti, Verani, 2015], [A., Cangiani, Collis, Dong, Georgoulis, Giani, Houston, 2016], [Cangiani, Dong, Geourgolis, 2016]}]
Stability and semi-discrete error estimates

Let $\alpha$ be large enough and let $u^h \in V_{hp}$ be the approximate solution obtained with a displacement DG formulation. Then,

$$\|u^h(t)\|_{E}^2 \leq \|u^h(0)\|_{E} + \int_{0}^{t} \rho^{-1}_{\ast} \|f(\tau)\|_{L^2(\Omega)} d\tau \quad 0 < t \leq T$$

Assume that $\alpha$ is large enough. Let $u^h$ be the approximated solution of $u$, assumed sufficiently regular. Then

$$\sup_{0 < s \leq T} \|u(s) - u^h(s)\|_{E} \lesssim \frac{h^{m-1}}{p^{k-3/2}}$$

where $m = \min(p + 1, k)$ with $k > 1 + d/2$. The hidden constant depends on $C(u, u_t, u_{tt}, T, \text{material properties})$.

$$\|u(s)\|_{E}^2 = \|\rho^{1/2} u_t(s)\|_{L^2(\Omega)}^2 + \|u(s)\|_{DG}^2$$
Time integration

\[
\begin{cases}
M\ddot{U} + A_{DG} U = F \\
U(0) = u_0 \\
\dot{U}(0) = u_1
\end{cases}
\]

For time discretizations we set \( \Delta t = T/N \), and \( t_n = n\Delta t \), for \( n = 0, \ldots, N \)

- **leap-frog** scheme
  - explicit, second-order accurate

- Runge-Kutta methods: **RK4** scheme
  - explicit, fourth-order accurate, \( \Delta t \) – adaptive

- **DG method of lines** [A., dal Santo, Mazzieri, Quarteroni, 2016]
  - implicit, arbitrary high-order accurate, \( \Delta p/\Delta t \) – adaptive
Stability of the leap-frog scheme

The leap-frog time integration scheme is stable provided that

\[ \Delta t \leq \frac{C_{CFL}(p^2)}{c_p} h \]

where \( c_p = \sqrt{(\lambda + 2\mu)/\rho} \) is the P-wave velocity.

Computed upper bound for \( C_{CFL} \)

<table>
<thead>
<tr>
<th>p</th>
<th>SEM</th>
<th>SIP</th>
<th>NIP</th>
</tr>
</thead>
<tbody>
<tr>
<td>2</td>
<td>0.3376</td>
<td>0.2621</td>
<td>0.2163</td>
</tr>
<tr>
<td>3</td>
<td>0.1967</td>
<td>0.1368</td>
<td>0.1045</td>
</tr>
<tr>
<td>4</td>
<td>0.1206</td>
<td>0.0795</td>
<td>0.0607</td>
</tr>
<tr>
<td>5</td>
<td>0.0827</td>
<td>0.0530</td>
<td>0.0400</td>
</tr>
<tr>
<td>6</td>
<td>0.0596</td>
<td>0.0374</td>
<td>0.0281</td>
</tr>
<tr>
<td>7</td>
<td>0.0449</td>
<td>0.0280</td>
<td>0.0210</td>
</tr>
<tr>
<td>8</td>
<td>0.0351</td>
<td>0.0216</td>
<td>0.0162</td>
</tr>
<tr>
<td>9</td>
<td>0.0281</td>
<td>0.0172</td>
<td>0.0129</td>
</tr>
<tr>
<td>10</td>
<td>0.0231</td>
<td>0.0140</td>
<td>0.0105</td>
</tr>
</tbody>
</table>

- The constants for the SIP are around 70% with respect the SEM
- The NIP has constants always more restrictive than those of SIP

\( p \)-rate: -1.8463, -1.9247, -1.9360
Convergence test (3D, hexahedral grids)

- $\Omega = (0, 1)^3$, $\lambda = \mu = \rho = 1$, time interval $(0, T] = (0, 10]$.
- Exact solution

$$u(t, x, y, z) = \sin(3\pi t) \begin{bmatrix} -\sin^2(\pi x) \sin(2\pi y) \sin(2\pi z) \\ \sin(2\pi x) \sin^2(\pi y) \sin(2\pi z) \\ \sin(2\pi x) \sin(2\pi y) \sin^2(\pi z) \end{bmatrix}.$$ 

- Leap-frog scheme used for the time integration with time step $\Delta t = 1 \cdot 10^{-5}$

Computed convergence rates

<table>
<thead>
<tr>
<th>$p$</th>
<th>SIP</th>
<th>SEM</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>1.1212</td>
<td>0.9492</td>
</tr>
<tr>
<td>2</td>
<td>2.1157</td>
<td>2.0622</td>
</tr>
<tr>
<td>3</td>
<td>2.8478</td>
<td>3.0135</td>
</tr>
<tr>
<td>4</td>
<td>3.7973</td>
<td>3.7973</td>
</tr>
</tbody>
</table>
Grid dispersion and dissipation errors

- New generalized eigenvalue approach
- Analysis for the fully-discrete approximations
- Cross-validation with SE discretization

- Low grid dispersion errors for $p \geq 4$ and $\delta < 0.2$ (5 or more points per wavelength)
- No-dissipation effects when using the leap-frog scheme

[De Basabe et al. 2010], [Ainsworth 2006], [Seriani et al. 2008]
Validation: Layer over halfspace

- $\Omega = (-30, 30) \times (-30, 30) \times (0, 17) \text{Km}$
- Seismic moment in $S = (0, 0, 2) \text{Km}$
- Receiver $R = (6, 8, 0) \text{Km}$

1 of 4 symmetric quadrants

Moment time history

<table>
<thead>
<tr>
<th>Layer</th>
<th>Depth [m]</th>
<th>$c_P$ [m/s]</th>
<th>$c_S$ [m/s]</th>
<th>$\rho$ [Kg/m$^3$]</th>
</tr>
</thead>
<tbody>
<tr>
<td>L</td>
<td>0-1000</td>
<td>4000</td>
<td>2000</td>
<td>2600</td>
</tr>
<tr>
<td>HS</td>
<td>1000-17000</td>
<td>6000</td>
<td>3464</td>
<td>2700</td>
</tr>
</tbody>
</table>

[Day et al. 2001]
Validation: Layer over halfspace (DGSEM vs SEM)

<table>
<thead>
<tr>
<th>Model</th>
<th>El.</th>
<th>p</th>
<th>d.o.f.</th>
<th>Δt</th>
</tr>
</thead>
<tbody>
<tr>
<td>SEM</td>
<td>814833</td>
<td>4</td>
<td>≈150 mln</td>
<td>3 \cdot 10^{-4}</td>
</tr>
<tr>
<td>SIP</td>
<td>70228</td>
<td>5</td>
<td>≈30 mln</td>
<td>5 \cdot 10^{-4}</td>
</tr>
</tbody>
</table>
Validation: Layer over halfspace (DGTSEM)

- Layer: $p = 5$
- Half-space: $p = 4$

Velocity recorded at point (6,8,0) Km

- Radial: $E = 0.034813$
- Transverse: $E = 0.034407$
Applications

- Emilia (IT) earthquake 29.05.2012, magnitude 6.0
- Christchurch (NZ) earthquake 22.02.2011, magnitude 6.2
- Acquasanta (IT) railway bridge
Emilia (IT) earthquake 29.05.2012, magnitude 6.0

Collapse of the San Francesco Church (XV century), Mirandola. Courtesy of A. Penna.

- Availability of an exceptional strong-motion dataset in near-fault conditions and on deep soft soil
- Good knowledge of a deep and large sedimentary basin such as the Po Plain
- Strong economic impact of the earthquake (∼13 billion of Euros, Munich RE)
- Previously selected as possible site for a Nuclear Power Plant in Italy due to its moderate seismicity

Structural map of Italy from [Bigi et al., 1992]. Epicenters of the two main shocks: May 20 ($M_W 6.1$) and May 29 ($M_W 6.0$).

[Bigi et al., 1992]
[Mazzieri et al., 2015]
Comparison between recorded (black line) and simulated (red line) three-component velocity waveforms for a representative subset of ten stations

- $p = 3$
- Dofs $\approx 350$ millions ($\approx 2$ millions elements)
- $\Delta t$ 0.001 s, $T = 30$ s
- $f_{\text{max}}$ 1.5 Hz
- Simulation time $\approx 5$ hours on 4096 cores (FERMI@CINECA)
Emilia (IT) earthquake 29.05.2012, magnitude 6.0

Map of permanent ground uplift simulated by SPEED (left) and observed by COSMO-Skymed InSAR processing (right)

[Mazzieri et al, 2015]
Emilia (IT) earthquake 29.05.2012, magnitude 6.0

Domain size: $74 \times 51 \times 20 \ km^3$

Hexahedral elements: $\approx 2 \ millions \ (\approx 500 \ millions \ of \ unknowns)$

Time step: $\Delta t = 5 \ 10^{-4}$ and final time $T = 50 \ s$.

Tests performed on FERMI Bluegene Q, CINECA Italy

[Dagna, P., Enabling SPEED for near Real-time Earthquakes Simulations, 2013]
Christchurch (NZ) earthquake 22.02.2011, magnitude 6.2

Geological map of Christchurch provided by GNS (New Zealand research institute) [Forsyth et al. 2008]

[Mazzieri et al. 2013]
Christchurch (NZ) earthquake 22.02.2011, magnitude 6.2

(a) Christchurch fault slip
Strike = 58°, Dip = 68° SE; Mw = 6.2

- basin (Vs = 300 m/s)
- basin (Vs = 1000 m/s)
- basin (Vs = 1500 m/s)
- volcano
- bedrock

~5m, N=1
~50m, N=2
~150m, N=4

DGSE methods for earthquake simulations
Séminaire du LJLL
Christchurch (NZ) earthquake 22.02.2011, magnitude 6.2

- Soft-soil $h \approx 25/50\text{m}, p = 2$
- Central Business District $h \approx 5\text{m}, p = 1$

<table>
<thead>
<tr>
<th>Layer</th>
<th>$c_S[\text{m/s}]$</th>
<th>$c_P[\text{m/s}]$</th>
<th>$\rho[\text{Kg/m}^3]$</th>
<th>$Q(\xi)$</th>
</tr>
</thead>
<tbody>
<tr>
<td>Foundation</td>
<td>400</td>
<td>650</td>
<td>2400</td>
<td>200</td>
</tr>
<tr>
<td>Building</td>
<td>100</td>
<td>163</td>
<td>2400</td>
<td>200</td>
</tr>
</tbody>
</table>

Mechanical parameters [Bielak et al. 2010]
Christchurch (NZ) earthquake 22.02.2011, magnitude 6.2

West-East (W-E) and South-North (S-N) Peak
Ground Velocity

Displacements on the buildings of the CBD
Acquasanta (IT) railway bridge

Acquasanta bridge and surrounding valley

Reference solution: SEM on a fine conforming grid with $p = 3$.

<table>
<thead>
<tr>
<th>Model</th>
<th>El.</th>
<th>$p$</th>
<th>d.o.f.</th>
<th>$\Delta t/\Delta t_{CFL}$</th>
</tr>
</thead>
<tbody>
<tr>
<td>SEM</td>
<td>38569</td>
<td>3</td>
<td>$\approx 3$ mln</td>
<td>2.3 %</td>
</tr>
<tr>
<td>SIP</td>
<td>20602</td>
<td>4-3-2</td>
<td>$\approx 1.8$ mln</td>
<td>6.3 %</td>
</tr>
</tbody>
</table>

[Mazzieri et al. 2013]
Acquasanta (IT) railway bridge

- Non-conforming mesh: different colors identify different sub-domains.
- Good fit for the data: grid dispersion errors are under 10%.

**Horizontal velocity in R**

**SEM - SIP**

[Mazzieri et al. 2013]
SPEED: Built-in capabilities http://speed.mox.polimi.it

1. Models
   - Elastic/visco-elastic soil models
   - Non-Linear Elastic soil model

2. Methods
   - Discontinuous Galerkin Spectral Element method
   - Leap-frog, Runge-Kutta (RK3 and RK4) time integration schemes
   - First order absorbing boundaries

3. Seismic excitation modes
   - plane wave load
   - Neumann surface load
   - volume force load

4. Pre and Post processing tools
   - Matlab GUI for input/output processing of earthquake scenarios
   - User Guide, Tutorials and documentation

5. Web-repository database of synthetic earthquake scenarios
SPEED: Available soon http://speed.mox.polimi.it

1 Models
   - Elasto-acoustic coupling

2 Methods
   - SEM-BEM coupling for absorbing boundaries
   - Discontinuous time integration schemes
   - Tetrahedral spectral elements
   - Polytopic grids

SPEED around the world (from 2013)

Academic Institutions (17) Companies (3)
Conclusions and further developments

Conclusions

- DG methods can be coupled with spectral element approximations in a "domain decomposition fashion" to deal efficiently with wave propagation phenomena in complex 3D configurations
- Exploit the flexibility of DG methods in tuning the discretization parameters while keeping limited the proliferation of the unknowns

Further developments

- 3D implementation of DG methods on polytopic grids and validation on real earthquake scenarios
- Fast solution techniques for TSEM
- High-order DG methods for time integration
Acknowledgments

SIR project n. RBSI14VT0S: PolyPDEs: Non-conforming polyhedral finite element methods for the approximation of partial differential equations.

Fondazione Cariplo/RL project n. 2015–0182 PolyNum: Polyhedral numerical methods for partial differential equations

MunichRe project: MRPMII: Spectral Element Methods for earthquake simulations (PI: Paolucci)

Thank you for your attention!!