On the discontinuous Galerkin finite element method for nonstationary convection-diffusion problems. Theory and applications to compressible flow[1]

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ABSTRACT

In this lecture we shall be concerned with several aspects of the numerical solution of convection-diffusion problems by the discontinuous Galerkin finite element method (DGFEM) and applications to compressible flow. The DGFEM is based on a piecewise polynomial approximation of the sought solution without any requirement on the continuity on interfaces between neighbouring elements. It is particularly convenient for the solution of conservation laws with discontinuous solutions or singularly perturbed convection-diffusion problems with dominating convection, when solutions contain very steep gradients.

The first subject is the analysis of error estimates of the DGFEM applied to the space semidiscretization of nonstationary convection-diffusion problems. We shall discuss the error estimates in $L^2(H^1)$- and $L^\infty(L^2)$-norm for linear and nonlinear problems, their optimality and uniformity with respect to the diffusion coefficient tending to zero. We shall also mention the space-time DG discretization. The theoretical results will be illustrated by numerical experiments.

In the second part, some applications of the DGFEM to the simulation of compressible flow, i.e. the solution of the compressible Euler and Navier-Stokes equations, will be presented. Our goal is to develop sufficiently accurate, efficient and robust numerical schemes allowing the solution of compressible flow for a wide range of Reynolds and Mach numbers. Our approach allows to solve gas flow with practically all Mach numbers (starting from Mach number $= 10^{-6}$ up to transonic regimes). The efficiency and accuracy of the method will be demonstrated by computational results.

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